Long-Run Neutrality and Superneutrality in an ARIMA Framework

By MARK E. FISHER AND JOHN J. SEATER*

We (i) formalize long-run neutrality (LRN) and long-run superneutrality (LRSN) in the context of a bivariate ARIMA model, (ii) show how the restrictions implied by LRN and LRSN depend on the orders of integration of the variables, (iii) apply our analysis to previous work, showing how that work is related to LRN and LRSN, and (iv) provide some new evidence on LRN and LRSN. (*JEL* E30)

In this paper, we formalize the "classical" concepts of long-run neutrality (LRN) and long-run superneutrality (LRSN) and derive testable implications. By LRN, we mean the proposition that permanent, exogenous changes to the *level* of the money supply ultimately leave the *level* of real variables and the nominal interest rate unchanged but ultimately lead to equiproportionate changes in the *level* of prices and other nominal variables; by LRSN, we mean the proposition that permanent, exogenous changes to the *growth rate* of the money supply ultimately lead to equal changes in the nominal interest rate and leave the *level* of real variables unchanged.[1] We show, among other things, that LRN is necessary but not sufficient for LRSN and that propositions about how permanent changes in the *growth rate* of money are ultimately reflected in the *growth rates* of other variables have LRN interpretations rather than LRSN interpretations.

Neither LRN nor LRSN refers to the short-run effects of money shocks; therefore, they differ from some recently developed concepts of neutrality in which the expected or perceived component of a money shock has no real effect at any time. These latter concepts, which are central to the rational-expectations literature on business cycles, should be distinguished from what we address in this paper.[2] Because LRN and LRSN do not depend on the short-run dynamics of the economy, structural details that are important for many issues are not relevant to LRN and LRSN. It is desirable, therefore, to have tests of LRN and LRSN that are relatively structure-free. A convenient setting for nonstructural tests is provided by a multivariate ARIMA model.

Our main result is that the restrictions implied by LRN and LRSN within an ARIMA framework depend on the orders of integration of both money and the other variable of interest. The orders of integration are important for two reasons. First, absent knowledge of the underlying struc-

*Board of Governors of the Federal Reserve System, Mail Stop 69, Washington, DC 20551, and Department of Economics, North Carolina State University, Raleigh, NC 27695, respectively. This research evolved from discussions between Fisher and Roger Kormendi. A substantial portion of this paper draws on Fisher (1988). We thank Stephen Blough, Peter Bloomfield, John Huizinga, Christopher Sims, Walter Thurman, and two referees for helpful comments on earlier drafts. The views expressed in this paper are those of the authors and do not necessarily represent those of the Board or the Federal Reserve System.

[1]Some writers (e.g., Don Patinkin, 1987; Bennett T. McCallum, 1990) exclude real balances from the list of real variables that do not change under superneutrality. We include real balances in the list because we derive implications that apply to *any* real variable: To exclude real balances explicitly would make our exposition cumbersome.

[2]Expectational neutrality concepts are considerably more difficult to formalize than the classical concepts (see Fisher and Seater, 1989).

ture, the consequences of an event cannot be inferred if the event has not occurred. In order for inferences regarding LRN (LRSN) to be drawn from a reduced form, the data must contain permanent stochastic changes in the level (growth rate) of the money supply. The second reason why the orders of integration are important is that the potential long-run response of one variable to another depends on their relative orders of integration; consequently, the parameter restrictions implied by LRN and LRSN depend on the difference between the orders of integration of the money supply and the other variable of interest. For example, if there are permanent stochastic changes in the level of the money supply (but not in its growth rate), then the parameter restriction implied by LRN depends on whether there are permanent stochastic changes in the level of output, changes in the growth rate of output, or no permanent stochastic changes in output at all.

The derivation of the appropriate testable restrictions constitutes the substance of this paper. Some of the results we obtain have been the subject of disputes in earlier literature. We are able to use our formalizations to reconcile those disputes. We also are able to reinterpret some of the empirical literature, finding that it supports both LRN and LRSN. We produce some new evidence of our own, with mixed results.

In Section I, we define LRN and LRSN in the context of an ARIMA framework and derive testable implications. In Section II, we discuss identification and estimation. In Section III, we review several papers from the past two decades to see how the findings can be interpreted in terms of LRN and LRSN, and in Section IV, we present our new evidence.

I. LRN and LRSN in an ARIMA Framework

To formalize our notions of LRN and LRSN and to derive the restrictions they imply, we first define the long-run derivative (LRD). Although the LRD can be defined without regard to a specific stochastic framework, we restrict attention here to a bivariate ARIMA model for expositional clarity.[3] We then define LRN and LRSN in terms of the LRD and describe the testable restrictions implied by the definitions.

A. *The Long-Run Derivative*

We assume a log-linear system of two variables with a stationary, invertible bivariate ARIMA representation.[4] Let m be the log of the nominal money supply M, and let y be an interest rate or the log of some other variable Y, such as the price level or real GNP. Let "m is $I(\gamma)$" stand for "m is integrated order γ," and let $\langle m \rangle$ represent the order of integration of m. For example, if m is $I(\gamma)$, then $\langle m \rangle = \gamma$. Let $\Delta \equiv (1-L)$. The growth rate of the money supply then is denoted by Δm, and $\langle \Delta m \rangle = \langle m \rangle - 1$. The autoregressive representation of the system is given by

$$(1) \quad a(L)\Delta^{\langle m \rangle} m_t = b(L)\Delta^{\langle y \rangle} y_t + u_t$$

$$d(L)\Delta^{\langle y \rangle} y_t = c(L)\Delta^{\langle m \rangle} m_t + w_t$$

where $a_0 = d_0 = 1$, and b_0 and c_0 are not restricted. The vector $(u_t \; w_t)'$ is assumed to be independently and identically distributed with mean zero and covariance Σ, the elements of which are σ_{uu}, σ_{uw}, and σ_{ww}. Constants and trends are suppressed; if a variable is stationary around a deterministic trend, we treat it as $I(0)$.[5]

Our formalizations of LRN and LRSN rely on conceptual experiments that focus on the extent to which m, Δm, y, and Δy are ultimately changed by an exogenous money-supply disturbance u. This treatment of u amounts to a structural assumption requiring an appropriate set of identifying restrictions. However, as we shall show, no

[3]See Fisher and Seater (1989) for a more general statement of the LRD.

[4]Cointegration and systems with more than two variables are discussed briefly in the Appendix. We do not treat fractional integration in this paper.

[5]Identification is discussed in the next section.

additional structure is required; in particular, the parameters in the distributed lags need not be structural. (We discuss identification further in Section II.) Because we deal with both levels and differences of variables, it is convenient to define the long-run derivative in terms of $x_t \equiv \Delta^i m_t$ and $z_t \equiv \Delta^j y_t$, where i and j equal 0 or 1.

The *long-run derivative of z with respect to a permanent change in x* is defined as follows: if $\lim_{k\to\infty} \partial x_{t+k}/\partial u_t \neq 0$, then

$$\text{LRD}_{z,x} \equiv \lim_{k\to\infty} \frac{\partial z_{t+k}/\partial u_t}{\partial x_{t+k}/\partial u_t}.$$

When $\lim_{k\to\infty} \partial x_{t+k}/\partial u_t = 0$, there are no permanent changes in the monetary variable and thus no long-run neutrality or superneutrality experiment to be examined; we simply leave the LRD undefined in this case. With this case set aside, the above defines the LRD as the limit of the ratio of two sequences. The sequence in the numerator measures the effect through time of an exogenous money disturbance on the variable z, and the sequence in the denominator measures the effect of the same money disturbance on the monetary variable x itself. Thus the limit of the ratio expresses the ultimate effect of a monetary disturbance on z relative to that disturbance's ultimate effect on x. Consider two examples in which y is the log of Y. First, when $z = y$, $x = m$, and $\langle y \rangle = \langle m \rangle = 1$, $\text{LRD}_{z,x}$ measures the long-run elasticity of Y with respect to M; second, when $z = y$, $x = \Delta m$, and $\langle y \rangle = \langle m \rangle = 2$, $\text{LRD}_{z,x}$ measures the long-run semielasticity of Y with respect to Δm. Our definitions of LRN and LRSN impose specific values for these LRD's.

To evaluate $\text{LRD}_{z,x}$, we use the impulse-response representation for x and z, which is given by the solution of (1):

$$(2) \quad x_t = \Delta^{-\langle x \rangle}[\alpha(L)u_t + \beta(L)w_t]$$

$$z_t = \Delta^{-\langle z \rangle}[\gamma(L)u_t + \lambda(L)w_t]$$

where

$$\alpha(L) = d(L)/[a(L)d(L) - b(L)c(L)]$$

$$\gamma(L) = c(L)/[a(L)d(L) - b(L)c(L)]$$

and so forth. From (2), $\partial x_{t+k}/\partial u_t$ equals the coefficient on L^k in $(1-L)^{-\langle x \rangle}\alpha(L)$. For example, when $\langle x \rangle = 0$, $\partial x_{t+k}/\partial u_t = \alpha_k$; when $\langle x \rangle = 1$, $\partial x_{t+k}/\partial u_t = \sum_{i=0}^{k}\alpha_i$. We use the following fact to evaluate the limits: $\lim_{k\to\infty} \zeta_k = \chi(1)$, where $\chi(L) \equiv (1-L)\zeta(L)$. Thus, $\lim_{k\to\infty} \partial x_{t+k}/\partial u_t = \Theta(1)$ where $\Theta(L) \equiv (1-L)^{1-\langle x \rangle}\alpha(L)$. Similarly, $\lim_{k\to\infty} \partial z_{t+k}/\partial u_t = \Gamma(1)$ where $\Gamma(L) \equiv (1-L)^{1-\langle z \rangle}\gamma(L)$.

The evaluation of $\text{LRD}_{z,x}$ depends on $\lim_{k\to\infty} \partial x_{t+k}/\partial u_t$, which in turn depends on $\langle x \rangle$: $\lim_{k\to\infty} \partial x_{t+k}/\partial u_t$ is zero, nonzero and finite, or infinite as $\langle x \rangle$ is less than, equal to, or greater than 1. When $\langle x \rangle = 0$, $\text{LRD}_{z,x}$ is undefined, as explained above. When $\langle x \rangle = 1$, $\text{LRD}_{z,x} = \Gamma(1)/\alpha(1)$, the ratio of the limits. When $\langle x \rangle > 1$, $\text{LRD}_{z,x}$ can be evaluated by differencing both sequences $\langle x \rangle - 1$ times,[6] which amounts to multiplying both $\Theta(L)$ and $\Gamma(L)$ by $(1-L)^{\langle x \rangle - 1}$ before evaluating the limit. Thus, when $\langle x \rangle \geq 1$, we can write

$$(3) \quad \text{LRD}_{z,x} = \frac{(1-L)^{\langle x \rangle - \langle z \rangle}\gamma(L)\big|_{L=1}}{\alpha(1)}.$$

Equation (3) shows that the value of $\text{LRD}_{z,x}$ depends on $\langle x \rangle - \langle z \rangle$. There are three cases we wish to consider. First, when $\langle x \rangle - \langle z \rangle \geq 1$, $\text{LRD}_{z,x} \equiv 0$. Second, when $\langle x \rangle - \langle z \rangle = 0$, $\text{LRD}_{z,x} = \gamma(1)/\alpha(1) = c(1)/d(1)$. Third, when $\langle x \rangle - \langle z \rangle = -1$, $\text{LRD}_{z,x}$ is finite only if $\gamma(1) = 0$, in which case $\text{LRD}_{z,x} = (1-L)^{-1}\gamma(L)/\alpha(1) = c^*(1)/d(1)$, where $c^*(L) \equiv (1-L)^{-1}c(L)$. Note that $c^*(1)$ is the sum of partial sums, $\sum_{j=0}^{\infty}\sum_{i=0}^{j}c_i$.[7] Table 1 summarizes the results of this subsection.

[6] This result follows from the discrete version of L'Hôpital's rule; see Konrad Knopp (1956).

[7] A similar analysis holds when $\langle x \rangle - \langle z \rangle < -1$.

TABLE 1—THE LONG-RUN DERIVATIVE

When:	$LRD_{z,x}$
$\langle x \rangle < 1$	is undefined
$\langle x \rangle - \langle z \rangle > 0$	$\equiv 0$
$\langle x \rangle - \langle z \rangle = 0$	$= c(1)/d(1)$
$\langle x \rangle - \langle z \rangle = -1$	$= c^*(1)/d(1)$

B. *Long-Run Neutrality*

We now use the foregoing material to define LRN.

Definition: Money is long-run neutral if $LRD_{y,m} = \lambda$, where $\lambda = 1$ when y is a nominal variable and $\lambda = 0$ when y is a real variable or the nominal interest rate.

We discuss the implications of $LRD_{y,m} = \lambda$ for system (1) for four cases.

Case (i).—When $\langle m \rangle < 1$, $LRD_{y,m}$ is not defined. There are no permanent stochastic changes in m, so LRN is not addressable.

There is an exception to the general statement that reduced-form testing of LRN has no content when there are no permanent changes in m. If $\langle m \rangle = 0$, $\langle y \rangle = 1$, and $c(1) \neq 0$, then transitory changes in m have permanent effects on y. Even though the LRD is not defined in this case, such a result certainly violates long-run neutrality. However, we do not believe that this exception vitiates the powerful insight of the general statement.

Case (ii).—When $\langle m \rangle \geq \langle y \rangle + 1 \geq 1$, $LRD_{y,m} \equiv 0$. In this case, the parameters of (1) (other than $\langle m \rangle$ and $\langle y \rangle$) are uninformative with respect to LRN.[8] The determination of LRN is immediate: If y is a real variable or the nominal interest rate, LRN holds; if y is a nominal variable, LRN is violated. The intuition is straightforward. When $\langle m \rangle = 1$ and $\langle y \rangle = 0$, permanent changes in m cannot be associated with permanent changes in y because the latter do not exist.

Recall, however, that the long-run derivative is defined in terms of *exogenous* money-supply disturbances. Suppose, instead, the monetary authority were targeting the price level in the presence of permanent changes in money demand. Under these conditions, permanent disturbances to the money supply would be induced by incipient permanent changes to the price level via the permanent changes in money demand. The money supply would be $I(1)$, and the price level would be $I(0)$. Yet this would not constitute evidence against LRN. A similar caveat is appropriate when we turn to LRSN.

Case (iii).—When $\langle m \rangle = \langle y \rangle \geq 1$, LRN implies

$$c(1)/d(1) = \lambda. \tag{4}$$

This case is of considerable importance. Some contributions to the literature suggest that (4) is not an implication of LRN and, to support that claim, provide examples where $\langle m \rangle = 0$ (see e.g., Robert J. Barro, 1981; McCallum, 1984). We agree when $\langle m \rangle = 0$, for then there are no permanent changes in money to discuss.

When $\langle m \rangle = \langle y \rangle = 1$, tests of LRN are possible because there are permanent changes in both m and y. When $\langle m \rangle = \langle y \rangle = 2$, there are permanent changes in the growth rates of both m and y. In this case, equation (3) implies $LRD_{\Delta y, \Delta m} = LRD_{y,m}$. In other words, propositions about how a permanent change in the growth rate of money ultimately affects the growth rate of another variable can be directly translated into propositions about how a permanent change in the level of money ultimately affects the level of another variable. This direct translation means that growth-rate-to-growth-rate propositions are not LRSN propositions, even though they involve

[8] Recall that we are working in an ARIMA framework. In a fully structural model, the parameters would be informative.

changes in the growth rate of money; rather, they are equivalent to level-to-level LRN propositions.

Case (iv).—When $\langle m \rangle = \langle y \rangle - 1 \geq 1$, LRN implies

$$(5) \qquad c^*(1)/d(1) = \lambda$$

which in turn implies $c(1) = 0$. In order to interpret restriction (5), suppose $\langle m \rangle = 1$ and $\langle y \rangle = 2$. In that case, an exogenous money disturbance u not only has a permanent effect on the level of the monetary variable (because it is integrated order one), but also might have a permanent effect on the *growth rate* of y (because y is integrated order two). However, if $c(1) = 0$, then in fact u has no such effect, since, given $\langle \Delta y \rangle = 1$, $\mathrm{LRD}_{\Delta y, m} = c(1)/d(1)$. Thus, $c(1) = 0$ expresses the fact that a necessary condition for LRN in this case is that money not change the growth rate of y. Money disturbances still might affect the ultimate level of y, and (5) expresses the parameter restriction required for such an effect to be consistent with LRN.

C. *Long-Run Superneutrality*

We now turn to LRSN, which concerns the effects of permanent changes in the growth rate of money on the level of other variables.

Definition: Money is long-run superneutral if $\mathrm{LRD}_{y,\Delta m} = \mu$, where $\mu = 1$ when y is the nominal rate of interest and $\mu = 0$ when y is a real variable.

The definition of LRSN only applies to those variables y for which LRN implies $\mathrm{LRD}_{y,m} = 0$. We discuss the implications of $\mathrm{LRD}_{y,\Delta m} = \mu$ for system (1) for four cases that correspond to the cases we considered in the previous section.

Case (i).—When $\langle \Delta m \rangle < 1$ (i.e., when $\langle m \rangle < 2$), $\mathrm{LRD}_{y,\Delta m}$ is not defined: there are no permanent stochastic changes in the money growth rate Δm. LRSN is not addressable.[9]

Case (ii).—When $\langle \Delta m \rangle \geq \langle y \rangle + 1 \geq 1$ (i.e., when $\langle m \rangle \geq \langle y \rangle + 2 \geq 2$), $\mathrm{LRD}_{y,\Delta m} \equiv 0$. LRSN holds without regard to the parameters in $c(L)$.[10] The intuition for this result is the same as that given in case (ii) above: When $\langle m \rangle = 2$ and $\langle y \rangle = 0$, permanent changes in Δm cannot be associated with nonexistent permanent changes in y.

Case (iii).—When $\langle \Delta m \rangle = \langle y \rangle \geq 1$ (i.e., when $\langle m \rangle = \langle y \rangle + 1 \geq 2$), LRSN implies

$$(6) \qquad c(1)/d(1) = \mu.$$

In this case, even though LRSN is falsifiable, LRN cannot be rejected, because $\mathrm{LRD}_{y,m} \equiv 0$.

Case (iv).—When $\langle \Delta m \rangle = \langle y \rangle - 1 \geq 1$ (i.e., when $\langle m \rangle = \langle y \rangle \geq 2$), LRSN implies

$$(7) \qquad c^*(1)/d(1) = \mu$$

which in turn implies $c(1) = 0$. In this case, $c(1) = 0$ is equivalent to the LRN of money with respect to y, which is falsifiable. LRN, which can be expressed as $\mathrm{LRD}_{\Delta y, \Delta m} = 0$ in this case, asserts that the growth rate of y is left unchanged in the long run. LRN is necessary for LRSN: If LRN does not hold, then LRSN cannot hold. However, if LRN holds, then LRSN requires that the sum of partial sums in the numerator takes on the value that ensures that the level of y adjusts appropriately. Table 2 summarizes the LRN and LRSN restrictions.

[9] The exception stated in case (i) of the previous section, appropriately modified, is applicable here as well.

[10] However, see the discussion in case (ii) of the previous section.

TABLE 2—LONG-RUN NEUTRALITY AND SUPERNEUTRALITY RESTRICTIONS

	$LRD_{y,m}$ LRN $\Rightarrow LRD_{y,m} = \lambda$			$LRD_{y,\Delta m}$ LRSN $\Rightarrow LRD_{y,\Delta m} = \mu$		
$\langle y \rangle$	$\langle m \rangle = 0$	$\langle m \rangle = 1$	$\langle m \rangle = 2$	$\langle m \rangle = 0$	$\langle m \rangle = 1$	$\langle m \rangle = 2$
0	undefined	$\equiv 0$	$\equiv 0$	undefined	undefined	$\equiv 0$
1	undefined	$c(1)/d(1)$	$\equiv 0$	undefined	undefined	$c(1)/d(1)$
2	undefined	$c^*(1)/d(1)$	$c(1)/d(1)$	undefined	undefined	$c^*(1)/d(1)$

II. Identification and Estimation

All of the restrictions we derived in the previous section can be written in one of the following ways:

$$c(1) - \pi d(1) = 0 \tag{8}$$

or

$$c^*(1) - \pi d(1) = 0 \tag{9}$$

where $\pi = 1$ or 0. These restrictions involve only parameters in the second equation of system (1), reproduced here:

$$d(L)\Delta^{\langle y \rangle} y_t = c(L)\Delta^{\langle m \rangle} m_t + w_t. \tag{10}$$

Under either of two recursive identification schemes, ordinary least squares (OLS) will consistently estimate the parameters in (10), which can be used to test (8) and (9). The first scheme imposes the identifying restrictions $c_0 = \sigma_{uw} = 0$, in which case the current value of the monetary variable, $\Delta^{\langle m \rangle} m_t$, does not enter (10). This scheme would be appropriate if, for example, y were real output and did not respond to a change in m during the current period because the measurement period was relatively short. The other recursive scheme imposes the identifying restrictions $b_0 = \sigma_{uw} = 0$, in which case $\Delta^{\langle m \rangle} m_t$ is predetermined in (10).

Restriction (9) implies $c(1) = 0$, in which case $c(L) = (1 - L)c^*(L)$. Conditional on $c(1) = 0$, we can rewrite (10) as

$$d(L)\Delta^{\langle y \rangle} y_t = c^*(L)\Delta^{\langle m \rangle + 1} m_t + w_t. \tag{11}$$

Equation (11) can be estimated by OLS, and a test can be performed on restriction (9).

It is possible, however, that neither recursive identification scheme is acceptable. For example, if y were the log of velocity, $\Delta^{\langle y \rangle} y_t$ would respond to $\Delta^{\langle m \rangle} m_t$ unless nominal income adjusted fully within the measurement period (which probably would require that the period be relatively long); in other words, we would expect $c_0 < 0$. At the same time, an increase in nominal income (from a positive w_t shock, holding u_t fixed) may lead the banking system to supply more money independently of any action the central bank may take. The money supply would be endogenous, at least in the short run, with $b_0 > 0$, even though u_t remained exogenous. In this example, the system could be identified with $\sigma_{uw} = 0$ and $c_0 = -1$, under the assumption that nominal income did not respond to money in the current period.

Regardless of the suitability of the recursive identification schemes, the individual parameters in $c(L)$ and $d(L)$ are not of interest: They are not structural. We are interested only in $c(1)/d(1)$ or $c^*(1)/d(1)$, which can be estimated directly in the frequency domain. Under certain conditions, the frequency-zero regression coefficient equals the appropriate LRD. In particular, when

$$b(1) = \sigma_{uw} = 0 \tag{12}$$

the frequency-zero regression coefficient in the regression of $\Delta^{\langle y \rangle} y$ on $\Delta^{\langle m \rangle} m$ equals

$c(1)/d(1)$.[11] Restriction (12) can be interpreted as asserting the "long-run exogeneity" of m, in the sense that a permanent change in y has no effect on m in the long run. Restriction (12) allows for both c_0 and b_0 to be nonzero, trading the assumption of predeterminedness for the assumption of long-run exogeneity. Similarly, when

$$(13) \qquad b^*(1) = \sigma_{uw} = 0$$

where $b^*(L) \equiv (1-L)^{-1}b(L)$, the frequency-zero regression coefficient in the regression of $\Delta^{\langle y \rangle}y$ on $\Delta^{\langle m \rangle +1}m$ equals $c^*(1)/d(1)$.[12]

III. Previous Related Work

Here we apply our analysis to some previous work, showing how it relates to LRN and LRSN.

A. *General Theory*

McCallum (1984) demonstrates two important points regarding LRN testing that we have addressed above. In his "second example," he shows that restriction (4) does not hold when $\langle m \rangle = 0$ but that it does hold when $\langle m \rangle = 1$. In his "third example," he notes that if one estimates equation (10) without including $\Delta^{\langle m \rangle}m_t$ when, in fact, $c_0 \neq 0$, then one may incorrectly estimate restriction (4) and falsely reject LRN. However, McCallum (1984 p. 13) draws the following conclusion from the three examples[13] in his paper:

> The foregoing examples should be sufficient to demonstrate that it is not generally appropriate to rely upon low-frequency measures of relationships among variables as indicators of the validity of propositions concerning "long-run" effects or relationships. The reason for this failure is basically the same in all of the examples: the low-frequency measures in question are simply not designed to reflect the distinction between anticipated and unanticipated fluctuations that is crucial for accurately characterizing intervariable relationships in many dynamic models.

We disagree. The distinction between anticipated and unanticipated fluctuations is not the source of the problems that McCallum uncovered in his second and third examples. Subject to the caveats laid out in Sections I and II (a sufficient order of integration and appropriate identification) the association of low-frequency times-series statistics with LRN and LRSN propositions *is* warranted, regardless of the presence of expectational relationships.

B. *Time-Domain Tests*

Leonall C. Andersen and Denis S. Karnosky (1972) provide an early example of correctly specified LRN tests. They estimate equations of the following log-linear form:

$$\Delta y_t = c(L)\,\Delta m_t + e(L)\,\Delta f_t + w_t$$

where m is the money supply, f is high-employment government expenditure, and y is either the price level or real output. They assert that LRN requires $c(1) = 1$ when

[11] Let $[\alpha(L)\ \beta(L)]$ and $[\gamma(L)\ \lambda(L)]$ be the rows of $\mathbf{H}(L)$, a 2×2 matrix. Then the covariance generating function for $(\Delta^{\langle m \rangle}m_t\ \ \Delta^{\langle y \rangle}y_t)'$ is given by $M(z) \equiv \mathbf{H}(z)\boldsymbol{\Sigma}\mathbf{H}(z^{-1})'$, where z is a complex variable. The frequency-zero regression coefficient in the "regression" of $\Delta^{\langle y \rangle}y$ on $\Delta^{\langle m \rangle}m$ is given by $M_{21}(1)/M_{11}(1)$, which equals $c(1)/d(1)$ when (12) holds.

[12] The covariance generating function for $(\Delta^{\langle m+1 \rangle}m_t\ \ \Delta^{\langle y \rangle}y_t)'$ is given by $\mathbf{N}(z) \equiv \mathbf{K}(z)\mathbf{M}(z)\mathbf{K}(z^{-1})'$, where rows of $\mathbf{K}(L)$ are $[\Delta\ 0]$ and $[0\ 1]$, and $\mathbf{M}(z)$ is given in footnote 11. The frequency-zero regression coefficient of $\Delta^{\langle y \rangle}y$ on $\Delta^{\langle m \rangle +1}m$ is given by $N_{21}(1)/N_{11}(1)$, which equals $c^*(1)/d(1)$ when (13) holds.

[13] In his "first example," McCallum (1984) shows that $c(1)/d(1) = 1$ is not an implication of the Fisher equation when the changes in expected inflation are temporary.

y is the price level and $c(1)=0$ when y is real output, which is correct under their null hypothesis that $e(1)=0$.[14] Their results are consistent with LRN.

Roger C. Kormendi and Philip G. Meguire (1984) explicitly recognize the relationship between the order of integration and the testability of LRN.[15] They estimate their equations in growth rates but discuss LRN in terms of levels. For each of 47 countries, they estimate system (1), where y is real output. They find $\langle y \rangle = 1$ for all 47 countries, $\langle m \rangle = 1$ for 43 countries, and $\langle m \rangle = 2$ for four countries. They assume $b(L) = \sigma_{uw} = 0$, and using a two-step procedure similar to Barro (1978), they estimate the following:[16]

$$d(L)\,\Delta y_t = [c(L)/a(L)]u_t + w_t.$$

They test the restriction $c(1)/a(1)=0$, which is a test of LRN for the countries where $\langle m \rangle = 1$.[17] However, for the countries where $\langle m \rangle = 2$, $c(1)/a(1)=0$ is an implication of LRSN. At the 10-percent significance level, Kormendi and Meguire reject $c(1)/a(1)=0$ for four of the 43 countries where $\langle m \rangle = 1$ and for none of the countries where $\langle m \rangle = 2$—evidence consistent with both LRN and LRSN.

C. *Frequency-Domain Tests*

In this subsection, we review two papers that present evidence using the frequency domain—Robert E. Lucas, Jr. (1980) and John Geweke (1986)—to see if nonstructural, reduced-form inferences regarding LRN and LRSN can be drawn from that evidence. The interpretation of the tests in these papers is more complicated than that of the time-domain tests discussed in the previous subsection. We find that some of the tests are not informative (from a reduced-form standpoint) regarding LRN or LRSN because the variables appear to have insufficient orders of integration.

Lucas (1980) examines two "quantity-theoretic" propositions relating money growth to inflation and interest rates. The propositions are "...that a given change in the rate of change of the quantity of money induces (i) an equal change in the rate of price inflation and (ii) an equal change in nominal rates of interest" (p. 1005). Lucas employs data on the money supply (M), the price level (P), and a nominal interest rate (R). He applies a two-sided, exponentially weighted, moving-average filter to Δm, Δp, and R and then examines the slopes of scatterplots of the filtered data, one for filtered Δp versus filtered Δm and another for filtered R versus filtered Δm. Lucas argues that the quantity-theoretic propositions are supported because the scatterplots appear to have slopes of 1. Charles H. Whiteman (1984) and McCallum (1984) separately show that Lucas's technique amounts to estimating the frequency-zero regression coefficient. We can ask how Lucas's evidence relates to LRN and LRSN under the assumption that the money supply is long-run exogenous, a condition that equates the frequency-zero regression coefficient with the LRD. Because Lucas does not discuss the order of integration of the variables, we examine several cases.

The slope of an OLS line through the data in the scatterplot of filtered Δp versus filtered Δm can be interpreted as an estimate of $\text{LRD}_{p,m}$ if $\langle m \rangle = \langle p \rangle = 1$, in which case Lucas's finding is consistent with LRN. The same conclusion holds if $\langle m \rangle = \langle p \rangle = 2$ and Δm is cointegrated with Δp. If, however, $\langle m \rangle = \langle p \rangle = 2$, but Δm is not cointegrated with Δp, then there is a nonstationary disturbance term in the relationship

[14] See Fisher (1988) for a discussion of LRD's when there are more than two variables.

[15] The central issue in Kormendi and Meguire's (1984) paper is the proposition that the magnitude of the short-run effects of monetary shocks on real output is negatively related to the variability of such shocks across regimes.

[16] Kormendi and Meguire (1984) mistakenly claim that the impact of u_t on y_{t+k} is captured by the coefficients in $c(L)/a(L)$ rather than in $c(L)/[a(L)d(L)]$. As a result, they miscalculate their variable χ for the 14 countries for which $d(L) \neq 1$, although the correctly calculated variables are not substantively different from the ones reported in their paper.

[17] Although $c(1)/a(1)$ does not equal $\text{LRD}_{y,m}$, $c(1)/a(1)=0$ if and only if $\text{LRD}_{y,m}=0$.

between filtered Δm and filtered Δp that makes inference about LRN based on Lucas's calculations problematic. Differencing the variables a second time before filtering would eliminate this problem.

With regard to Lucas's second scatterplot, as long as $\langle m \rangle > \langle R \rangle$, the slope conveys no direct information about LRN since, $\text{LRD}_{R,m} \equiv 0$. Furthermore, the slope conveys no information about LRSN unless $\langle m \rangle \geq 2$, since $\text{LRD}_{R,\Delta m}$ would be undefined otherwise. However, suppose that $\langle m \rangle = 2$ and $\langle R \rangle = 1$ and further suppose that Δm and R are cointegrated. There would be permanent changes both in the growth rate of money and in the nominal interest rate, and the disturbance term would be stationary. Under these conditions, the scatterplot slope of filtered R versus filtered Δm would measure $\text{LRD}_{R,\Delta m} = c(1)/d(1)$, and Lucas's finding that this slope equals 1 is consistent with LRSN.

Whiteman (1984) presents a useful critique of inferences that are based on Lucas's calculations. He analyzes a model that can display the Mundell-Tobin effect, the "neoclassical growth model" in Lucas (1975). Because he treats the money supply as exogenous. we can equate the sum of coefficients he calculates with the LRD. He asserts that Lucas's "...calculations do not properly pit the quantity theory against the Mundell-Tobin effect" (p. 748) and that, in fact Lucas's "...results can be interpreted as evidence in favor of the effect" (p. 743). When the Mundell-Tobin effect is present, a permanent increase in the growth rate of money reduces the real rate of interest, and thus the nominal rate of interest rises by less than the increase in the growth rate of money. Clearly, the presence of the Mundell-Tobin effect would violate LRSN. It has no bearing on LRN, however.

Whiteman (1984) shows, regarding the slope of Lucas's first scatterplot, that $\text{LRD}_{p,m} = 1$ as long as $\langle m \rangle = \langle p \rangle \geq 1$, so that LRN holds in the model irrespective of the Mundell-Tobin effect. Regarding the slope of Lucas's second scatterplot, Whiteman shows that when $\langle m \rangle = 2$ and $\langle R \rangle = 1$, $\text{LRD}_{R,\Delta m} = 1$ only if there is no Mundell-Tobin effect in the model. Under those conditions, a test of LRSN can be based on the slope of the second scatterplot (ignoring the possible nonstationary disturbance problem). However, Whiteman argues that $\langle m \rangle = 1$. Because $\text{LRD}_{R,\Delta m}$ is undefined when $\langle m \rangle = 1$, we agree with Whiteman that Lucas's "calculations do not properly pit the quantity theory against the Mundell-Tobin effect" in that case. Whiteman (1984) shows that, given the structure of the model, $\langle m \rangle = 1$ and $c(1)/d(1) = 1$ together imply the presence of the Mundell-Tobin effect.

In summary, Lucas's evidence appears to be consistent with the LRN of money with respect to prices, but it is uninformative (from a reduced-form standpoint) regarding the LRSN of money with respect to the nominal interest rate.

In an innovative paper, Geweke (1986) defines "structural neutrality" and applies his decomposition of feedback by frequency (Geweke, 1982) to test the long-run superneutrality of money with respect to output, real rates, real balances, and velocity. Geweke defines structural neutrality in a system that can have more than two variables, and some of the tests he conducts involve more than two variables. However, for our purposes, nothing is lost by restricting the discussion to two variables. Therefore, we cast our discussion of Geweke's paper in terms of equation (1). We discuss three aspects of Geweke's paper: the definition of structural neutrality, the relationship between the tests Geweke conducts and LRN and LRSN, and the empirical findings.

Geweke (1986 p. 2) defines structural neutrality as follows: "We shall say that the variables $\underline{x}_t$ are structurally neutral with respect to $\underline{y}_t$ if $H^*(1) = 0$, that is, the total multiplier in $\underline{x}_t$ with respect to $\underline{y}_t$ is zero." In terms of equation (1), $\underline{x}_t = \Delta^{\langle m \rangle} m_t$, $\underline{y}_t = \Delta^{\langle y \rangle} y_t$, and $H^*(1) = c(1)$. Geweke uses his measure of feedback from $\Delta^{\langle m \rangle} m$ to $\Delta^{\langle y \rangle} y$ at frequency zero, which equals zero when $c(1) = 0$, to test for structural neutrality. The restriction implied by structural neutrality is the same as our LRN restriction for real variables and the nominal interest rate when

$\langle m \rangle = \langle y \rangle \geq 1$; it is also the same as our LRSN restriction for real variables when $\langle \Delta m \rangle = \langle y \rangle \geq 1$.[18]

The total multiplier comes from the so-called final form, the solution of the second equation in (1) for $\Delta^{\langle z \rangle} z_t$:

$$\Delta^{\langle z \rangle} z_t = [c(L)/d(L)]\Delta^{\langle x \rangle} x_t + w_t/d(L).$$

The total multiplier equals $c(1)/d(1)$, which is algebraically equivalent to $\text{LRD}_{z,x}$ when $\langle x \rangle = \langle z \rangle \geq 1$. Conceptually, however, the standard interpretation of the total multiplier is distinct from the LRD. Whereas the LRD measures how the permanent changes in x_t *that are in the data* affect z_t, the total multiplier measures how a change in $E[\Delta^{\langle x \rangle} x_t]$ *that is not in the data* would affect $E[\Delta^{\langle z \rangle} z_t]$ if such a change were to occur. For example, when $\langle m \rangle = \langle y \rangle = 0$, $\text{LRD}_{y,m}$ is not defined, because no permanent changes are in the data; nonetheless, by the standard interpretation of the total multiplier, one can infer what would happen if the mean of m were changed. Such inference is justified, of course, only if $c(1)/d(1)$ is invariant with respect to interventions in $E[\Delta^{\langle m \rangle} m_t]$.

Thus, our definitions of LRN and LRSN are fundamentally different from Geweke's (1986) definition of structural neutrality. In Geweke's definition, the variables of interest are the stationary variables: the variables $\Delta^{\langle m \rangle} m_t$ and $\Delta^{\langle y \rangle} y_t$, rather than m_t (or Δm_t) and y_t as in our definition. To highlight the difference, consider either McCallum's (1984) "second example" or the rational-expectations solution to the Cagan money-demand schedule in Whiteman (1987).[19] The models in these examples display LRN in the face of permanent changes in the money supply, but when $\langle m \rangle = 0$ and y is real balances, $c(1) \neq 0$.[20] According to Geweke's (1986) definition, money is not structurally neutral in those models when it is stationary. In contrast, our definition tells us we can make no reduced-form inferences regarding LRN or LRSN when money is stationary.

Now consider the informativeness of Geweke's (1986) tests regarding LRN and LRSN from a reduced-form standpoint. Geweke treats the nominal money supply as $I(1)$. Given this order of integration, only LRN is testable; LRSN is not. Consequently, we interpret Geweke's (1986) tests as reduced-form tests of LRN. The other variables Geweke deals with are real money balances, real output, and velocity [all of which he treats as $I(1)$] and real interest rates [which he treats as $I(0)$]. For real rates, therefore, not even LRN is testable since $\text{LRD}_{R,m} \equiv 0$ regardless of the value of $c(1)$.[21]

Geweke's (1986) empirical results provide strong support for the LRN of money with respect to output but substantial evidence against the LRN of money with respect to velocity (and, to a lesser extent, real balances). The results regarding velocity are puzzling regardless of whether one interprets them in terms of neutrality or superneutrality: why should a change in the level (growth rate) of money lead to a change in the level (growth rate) of velocity? This finding may be caused by the identification scheme Geweke (1986) uses. He imposes recursivity, setting $c_0 = \sigma_{uw} = 0$. This scheme may be innocuous when y is real output. However, when y is velocity, $c_0 = 0$ is tantamount to assuming that nominal income fully responds to the money supply within the current period. (Moreover, when combined with the restriction that money has no impact on real output in the current period, the identification scheme asserts that prices adjust fully.) Thus Geweke's (1986) identification scheme imposes the neutrality of

[18]Although structural neutrality is applicable only to real variables y, the relationship between money and nominal income, for example, can be converted into a relationship between money and velocity.

[19]Exercise 11 in Chapter XI.

[20]In these models, a temporary increase in the money supply induces a temporary increase in the price level that signals a temporary decrease in the expected cost of holding money, with the result that real balances increase temporarily.

[21]When $\langle m \rangle = 1$ and $\langle R \rangle = 0$, $c(1)/d(1)$ measures the LRD of money with respect to the *integral* of R, which is $I(1)$ by construction. We do not have a useful interpretation for $\text{LRD}_{\int R,m}$.

money with respect to nominal income in the very short run but allows money to have a nonneutral impact in the long run. As noted in Section II, $c_0 = -1$ would seem to be a more natural restriction than $c_0 = 0$ when y is velocity.[22] Alternatively, there is the identification scheme of long-run exogeneity which does not restrict either c_0 or b_0.

In summary, we interpret Geweke's (1986) results as (i) uninformative regarding LRSN; (ii) providing support for the LRN of money with respect to output; (iii) uninformative regarding the LRN of money with respect to the real interest rate; and (iv) uninformative (because of possible misspecification) regarding the LRN of money with respect to velocity and real balances.

IV. Additional Evidence

We have conducted two tests of LRN and LRSN that rely on the assumption of the long-run exogeneity of money.[23] (Recall from Section II that the frequency-zero regression coefficient in the regression of $\Delta^{\langle y\rangle}y$ on $\Delta^{\langle m\rangle}m$ equals $c(1)/d(1)$ when $b(1) = \sigma_{uw} = 0$.) We estimated $c(1)/d(1)$ using the Bartlett estimator of the frequency-zero regression coefficient, which can be calculated using moving averages of the observations.[24] This estimator is given by $\lim_{k\to\infty} b_k$, where b_k is the slope coefficient from the regression

$$(14)\quad \left[\sum_{j=0}^{k} \Delta^{\langle y\rangle} y_{t-j}\right] = a_k + b_k\left[\sum_{j=0}^{k} \Delta^{\langle m\rangle} m_{t-j}\right] + e_{kt}.$$

When $\langle y\rangle = \langle m\rangle = 1$, (14) becomes

$$(15)\quad (y_t - y_{t-k-1}) = a_k + b_k(m_t - m_{t-k-1}) + e_{kt}.$$

In this case, b_k is the slope of a scatterplot of y growth rates versus m growth rates, and the Bartlett estimator is the limit of that slope as the span over which those growth rates are computed goes to infinity. When $\langle y\rangle = 1$ and $\langle m\rangle = 2$, (14) can be written as

$$(16)\quad (y_t - y_{t-k-1}) = a_k + b_k(\Delta m_t - \Delta m_{t-k-1}) + e_{kt}$$

in which b_k has a similar interpretation.

The first of our tests uses Milton Friedman and Anna J. Schwartz's (1982) annual data for the United States over 1869–1975 for money, prices, nominal income, and real income taken from their table 4.8. The data are treated as a single regime. The variables appear to be $I(1)$ in their logs, so that the LRN restriction $c(1)/d(1) = \lambda$ is testable. Using (15), estimates of b_k were obtained for $k = 1$–30, and 95-percent confidence intervals corrected by Whitney K. Newey and Kenneth D. West's (1987) technique were constructed from a t distribution using $107/k$ degrees of freedom.

The neutrality results are mixed (see Figs. 1–3). The data support LRN with respect to

[22] The analogous restriction when y is real balances is $c_0 = 1$.

[23] These tests are also reported in Fisher (1988), where they are discussed in more detail.

[24] The Bartlett estimator smooths the periodogram using linearly decreasing weights (see M. B. Priestley, 1981). Our estimator can be seen to be the Bartlett estimator of the frequency-zero regression coefficient by writing the covariance of the moving averages of the observations in terms of the autocovariances:

$$\mathrm{Cov}\left(\sum_{i=0}^{k-1} x_{t-i}, \sum_{j=0}^{k-1} y_{t-j}\right) = \sum_{i=0}^{k-1}\sum_{j=0}^{k-1} \gamma(i-j) = k\gamma(0) + 2\sum_{i=1}^{k-1}(k-i)\gamma(i)$$

where $\gamma(i) = \mathrm{Cov}(x_{s+i}, y_s)$. A similar expression is obtained for the variance of a moving average. It is unnecessary to normalize the weights by dividing by k since k cancels from the numerator and denominator of the regression coefficient. See David R. Brillinger (1975) for the asymptotic distribution of estimators of frequency-dependent regression coefficients.

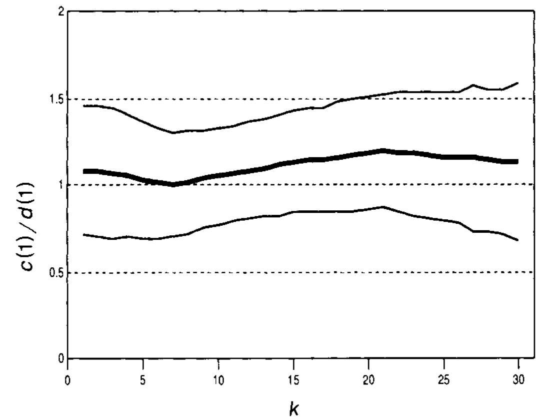

FIGURE 1. NOMINAL INCOME, U.S. DATA

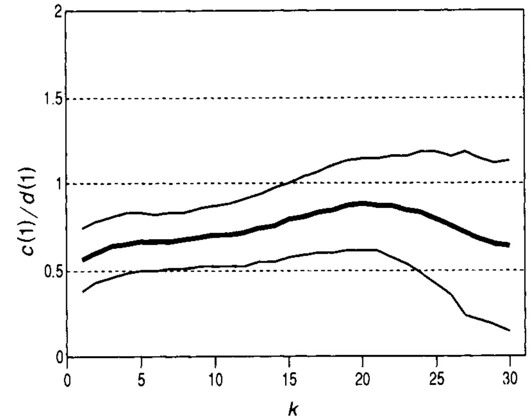

FIGURE 3. PRICES, U.S. DATA

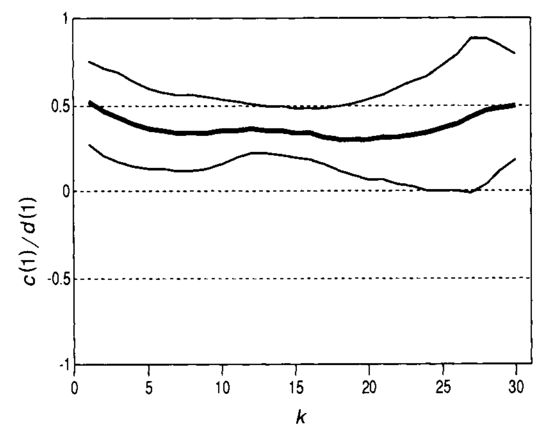

FIGURE 2. REAL INCOME, U.S. DATA

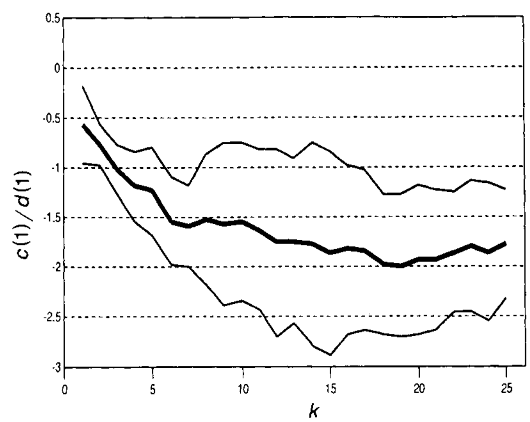

FIGURE 4. REAL BALANCES, GERMAN DATA

nominal income and prices. For nominal income, the 95-percent confidence interval around b_k includes 1 at all $k \leq 30$; for prices, it includes 1 for all $k \geq 15$. However, the lower bound of the confidence interval for real output lies above zero (except for $25 \leq k \leq 27$), which does not support LRN.

Our second test uses monthly data from the German hyperinflation following World War I.[25] Although the data contain only 55 observations, the rapid changes in the growth rates of money and prices over the sample may make the data useful for the analysis of long-run propositions. The money supply appears to be $I(2)$, whereas real balances appear to be $I(1)$. These orders of integration imply, first, that money is long-run neutral with respect to real balances (and to prices) and, second, that the LRSN restriction $c(1)/d(1)=\mu$ is testable. Using (16), estimates of b_k were obtained for $k =$ 1–25. The confidence intervals were corrected using the Newey-West technique and based on $55/k$ degrees of freedom.

The upper bound of the 95-percent confidence interval is less than zero at all lags, clearly implying that money is not superneutral with respect to real balances (see Fig. 4). The estimates suggest that a 1-

[25]Data are from Aris A. Protopapadakis (1979).

percentage-point permanent increase in the money-supply growth rate leads to about a 1.8-percent decrease in real balances. During the period studied, monthly continuously compounded money growth rates rose on the order of 200 percentage points, while real balances fell to about 3 percent of their original level. Notice that $e^{-0.018(200)} \approx 0.03$.

V. Conclusion

In this paper, we have formalized the classical notions of long-run neutrality and superneutrality in a bivariate log-linear ARIMA framework and derived testable implications for each concept. We have shown that the orders of integration of both the money stock and the other variable of interest are of primary importance in specifying the appropriate restrictions, thereby reconciling several strands of the literature and clearing up some disagreement concerning LRN and LRSN testing. In applying our analysis to several important contributions to the literature, we show that sometimes the evidence can be given interpretations different from those of the original authors. In particular, we interpret some of the tests of superneutrality as tests of neutrality and vice versa. Where the data appear sufficiently integrated to address LRN, they mostly support it. Our finding with respect to output in the United States is the exception. In two instances, the data are sufficiently integrated to address LRSN: Kormendi and Meguire (1984) provide evidence in favor of LRSN with respect to output, and we find evidence against LRSN with respect to real balances in the German hyperinflation.

Appendix[26]

Our formalizations of LRN and LRSN do not involve cointegration.[27] Cointegration plays no role because LRN and LRSN are based on how *changes* in money or its growth rate are ultimately related to *changes* in other variables. In general, cointegration is neither necessary nor sufficient for either LRN or LRSN. A full discussion of cointegration and its relationship to LRN and LRSN is beyond the scope of this paper. Instead we provide a simple example to show that cointegration per se does not affect the restrictions we derived.

Suppose $\langle m \rangle = \langle y \rangle = 1$, and let equation (1) be modified as follows:

$$\text{(A1)} \quad a(L)\,\Delta m_t = b(L)\,\Delta y_t + u_t$$
$$d(L)\,\Delta y_t = c(L)\,\Delta m_t + \Delta w_t .$$

If $c(1) \neq 0$, then m_t and y_t are cointegrated and $y_t - [c(1)/(d(1)]m_t$ is stationary. Our notion of LRN requires permanent, exogenous shocks to the money supply. In (A1) there is a single, exogenous source of nonstationarity, u_t; w_t does not contribute to the nonstationarity of the system. The LRN restriction $c(1)/d(1)$ remains appropriate. Note that the cointegration of m_t and y_t is sufficient to reject $c(1)/d(1) = 0$. In addition, if $\langle m \rangle = 2$, then LRSN would require the absence of cointegration between Δm and both Δy and y (except that if y were the nominal interest rate, Δm could be cointegrated with y).

When there are more than two variables, the LRD's may be different from the parameters in the cointegrating vectors, because the LRD's measure *mutatis mutandis* effects while the parameters in the cointegrating vectors measure *ceteris paribus* effects. The following example, in which $\langle m \rangle = \langle q \rangle = \langle p \rangle = 1$, illustrates this point:

$$\text{(A2)} \quad \Delta m_t = u_t$$
$$\Delta q_t = a\,\Delta m_t + w_t$$
$$\Delta p_t = b\,\Delta m_t + c\,\Delta q_t + \Delta \varepsilon_t$$

where $b > 0$ and $c < 0$. There is a single cointegrating relationship in (A2): $p_t - bm_t - cq_t$ is $I(0)$. In this example, $\text{LRD}_{p,m} = b + ac$. Thus, $\text{LRD}_{p,m} \neq b$ as long

[26]See Fisher (1988) for a more thorough discussion of the issues in this appendix.

[27]Robert F. Engle and Clive W. J. Granger (1987) discuss cointegration at length.

as $\text{LRD}_{q,m}(=a)\neq 0$, even though m and q are not cointegrated.

REFERENCES

Andersen, Leonall C. and Karnosky, Denis S., "The Appropriate Time Frame for Controlling Monetary Aggregates: The St. Louis Evidence," in *Controlling Monetary Aggregates II: The Implementation*, Conference Series No. 9, Boston: Federal Reserve Bank of Boston, 1972, pp. 147–77.

Barro, Robert J., "Unanticipated Money, Output, and the Price Level in the United States," *Journal of Political Economy*, August 1978, *86*, 549–80.

______, "The Equilibrium Approach to Business Cycles," in Robert J. Barro, ed., *Money, Expectations, and Business Cycles: Essays in Macroeconomics*, New York: Academic Press, 1981, pp. 41–78.

Brillinger, David R., *Time Series: Data Analysis and Theory*, New York: Holt, Rinehart and Winston, 1975.

Engle, Robert F. and Granger, Clive W. J., "Co-integration and Error Correction: Representation, Estimation, and Testing," *Econometrica*, March 1987, *55*, 251–76.

Fisher, Mark E., "The Time-Series Implications of the Long-Run Neutrality and Superneutrality of Money," Ph.D. dissertation, University of Chicago, 1988.

______ **and Seater, John J.,** "Neutralities of Money," Department of Economics and Business, unpublished manuscript, North Carolina State University, 1989.

Friedman, Milton and Schwartz, Anna J., *Monetary Trends in the United States and the United Kingdom*, Chicago: University of Chicago Press, 1982.

Geweke, John F., "Measurement of Linear Dependence and Feedback Between Multiple Time Series," *Journal of the American Statistical Association*, June 1982, *77*, 304–13.

______, "The Superneutrality of Money in the United States: An Interpretation of the Evidence," *Econometrica*, January 1986, *54*, 1–21.

Kormendi, Roger C. and Meguire, Philip G., "Cross-Regime Evidence of Macroeconomic Rationality," *Journal of Political Economy*, October 1984, *92*, 875–908.

Knopp, Konrad, *Infinite Sequences and Series*, [translated by Frederick Bagemihl], New York: Dover, 1956.

Lucas, Robert E., Jr.,, "An Equilibrium Model of the Business Cycle," *Journal of Political Economy*, December 1975, *83*, 1113–44.

______, "Two Illustrations of the Quantity Theory of Money," *American Economic Review*, December 1980, *70*, 1005–14.

McCallum, Bennett T., "On Low-Frequency Estimates of Long-Run Relationships in Macroeconomics," *Journal of Monetary Economics*, July 1984, *14*, 3–14.

______, "Inflation: Theory and Evidence," in Benjamin M. Friedman and Frank H. Hahn, eds., *Handbook of Monetary Economics*, Vol. 2, New York: North-Holland, 1990, pp. 963–1012.

Newey, Whitney K. and West, Kenneth D., "A Simple, Positive Semi-definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix," *Econometrica*, May 1987, *55*, 703–8.

Patinkin, Don, "Neutrality of Money," in John Eatwell, Murray Milgate, and Peter Newman, eds., *The New Palgrave: A Dictionary of Economics*, Vol. 3, New York: Stockton, 1987, pp. 639–45.

Priestley, M. B., *Spectral Analysis and Time Series*, New York: Academic Press, 1981.

Protopapadakis, Aris A., "Money, Prices, and Exchange Rates in the German Hyperinflation: An Expectations Approach," Ph.D. dissertation, University of Chicago, 1979.

Whiteman, Charles H., "Lucas on the Quantity Theory: Hypothesis Testing without Theory," *American Economic Review*, September 1984, *74*, 742–9.

______, *Problems in Macroeconomic Theory: Solutions to Exercises from Thomas J. Sargent's Macroeconomic Theory*, 2nd Ed., Boston: Academic Press, 1987.